lowed 3 dB to convert our double-sideband measurements to single-sideband for comparison. This may well be too large an increase since we are thereby assuming the mixer to respond equally in both sidebands, which is unlikely given the resonant nature of the diode–stripline–IF filter combination.

The local oscillator power requirement of our mixer is seen to be relatively low; this becomes important if the design is to be used at higher frequencies [10]. Fig. 10 shows that there is some residual IF mismatch at 3.9 GHz which could be removed by an appropriate IF impedance matching transformer leading to a further small improvement in performance.

The most significant advantage of the new design lies in the simplicity of the structure. Given the availability of diodes, a mixer with good performance can be assembled without sophisticated bonding or whiskering equipment. Furthermore, the reduction of all linear dimensions (including those of the Schottky barrier) by up to a factor of two would appear to present no difficulty either in diode processing or mixer circuit construction and assembly. We, therefore, anticipate that the same basic design can be used for operation up to 200 GHz.

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Scattering at an N-Furcated Parallel-Plate Waveguide Junction

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***Abstract*— Using the conservation of complex power technique (CCPT), this paper presents a solution to the problem of EM scattering at the junction of a parallel-plate waveguide and an N-furcated parallel-plate waveguide with arbitrarily spaced thick septa. Although this junction can be regarded as an $(N+1)$-port configuration, the problem is formulated so that it is viewed mathematically as a generalized 2-port. This leads to very simple expressions for the scattering parameters of the junction. Convergent numerical results are presented for bifurcated, trifurcated, and 4-furcated structures, and the effects of varying the thickness of the septa are investigated. The formulation is directly applicable to N-furcated rectangular waveguide junctions having TE_{n0} excitation, with application in the design of E-plane filters.**

I. Introduction

Electromagnetic scattering at the junction of a parallel-plate waveguide and a bifurcated parallel-plate waveguide with a septum of vanishing thickness has been studied by Mittra and Lee [1], who provided analytical solutions using the residue calculus method and the Wiener–Hopf technique. Moreover, a quasi-static solution using the singular integral equation method has been given by Lewin [2] for the case of a centrally located infinitely thin septum.

Trifurcated waveguide junctions were treated by Pace and Mittra [3], who considered the structure to be two bifurcated junctions in tandem; the overall solution was deduced with the help of the generalized scattering matrix technique [1].

The N-furcated junction has also been considered, in early papers, by Heins [4] and Igarashi [5]; however, their methods apply only to equally spaced thin septa.

In regard to bifurcated guides with thick septa, one may use the generalized scattering matrix technique, considering the junction as a bifurcated junction with a thin septum followed by a step discontinuity [6]. However, it would be very laborious to apply this technique repeatedly for the problem of an N-furcated waveguide junction with $N-1$ arbitrarily spaced thick septa.

In some recent papers [7]–[9], the conservation of the complex power technique (CCPT) has been used to obtain theoretically exact solutions with numerically convergent results to the problem of scattering at certain waveguide junctions. In this paper, the CCPT is applied to the specific case of the junction of a parallel-plate waveguide and an N-furcated parallel-plate waveguide, as shown in Fig. 1. The thicknesses $t_1, t_2, \cdots, t_{N-1}$ of the $N-1$ septa are not necessarily equal, nor are the separations between plates $L_1, L_2, L_3, \cdots, L_N$ of the N waveguides; the sole constraint is that $t_1 + t_2 + \cdots + t_{N-1} + L_1 + L_2 + \cdots + L_N = L$, where L is the separation between plates of the guide which forms the junction at $z=0$ with the N-furcated guide. Note also that the dielectric constant ϵ_{rn} in each waveguide is arbitrary.

Although in this contribution we only consider the problem of N-furcated parallel-plate waveguide junctions for TE_n and TM_n excitation, the formulation is also directly applicable to the

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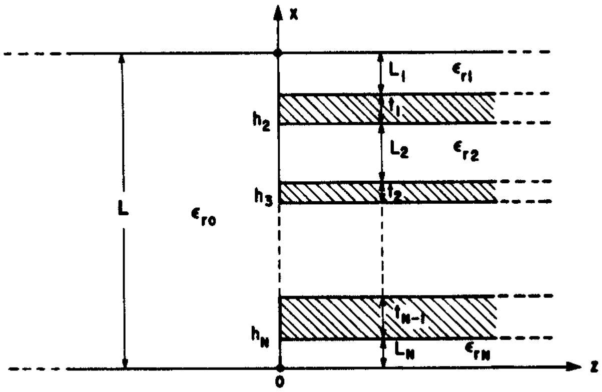

Fig. 1. An N-furcated parallel-plate waveguide junction.

problem of N-furcated rectangular waveguide junctions for TE_{n0} excitation. Moreover, the generalized scattering matrix technique [1, pp. 207–217] may be used to treat finite length septa. Thus, the present approach promises to be useful in the design of E-plane filters [6], [10], [11].

II. Formulation of the Problem

Although the N individual waveguides of the N-furcated structure are physically isolated by the $N-1$ perfectly conducting thick septa, it will nevertheless be possible to regard these N-ports as a generalized 1-port so that the junction at $z=0$ is simply a 2-port junction of a normal parallel-plate waveguide (for $z<0$) and an N-furcated parallel-plate waveguide (for $z>0$). Moreover, it was shown in [7], that if the electromagnetic fields in each waveguide are expanded in complete sets of orthogonal TE and TM modes, the scattering at such junctions involves no cross coupling between TE and TM modes. Accordingly, the junction fields can be expanded in terms of TE modes only for the case of TE excitation, or TM modes only if a TM field is incident on the junction.

Let the transverse E-fields (TE or TM as the case may be) in the ith guide at the junction plane $z=0$ be given by the modal series expansion[1]

$$\vec{E}_i(x)=\sum_l A_{il}\vec{e}_{il}(x)=\underline{\vec{e}}_i^T(x)\underline{A}_i \tag{1}$$

for $i=0,1,2,\cdots,N$.

In (1), A_{il} is the lth mode amplitude in the ith guide and $\vec{e}_{il}(x)$ is the modal vector field. As indicated in (1), the field in the ith guide can be succinctly represented as the product of the transpose of $\underline{\vec{e}}_i(x)$, a column matrix whose lth element is the vector field $\vec{e}_{il}(x)$, and of $\underline{A}_i$, a column matrix whose lth element is A_{il}.

At the junction $z=0$, the boundary conditions on the tangential E-field dictate that

$$\vec{E}_o(x)=\underline{\vec{e}}_o^T(x)\underline{A}_o=\begin{cases}\vec{E}_1(x)=\underline{\vec{e}}_1^T(x)\underline{A}_1 & \text{over } L_1\\ 0 & \text{over } t_1\\ \vec{E}_2(x)=\underline{\vec{e}}_2^T(x)\underline{A}_2 & \text{over } L_2\\ \vdots & \vdots\\ 0 & \text{over } t_{N-1}\\ \vec{E}_N(x)=\underline{\vec{e}}_N^T(x)\underline{A}_N & \text{over } L_N\end{cases} \tag{2}$$

[1]From now on we will use the following notations: T denotes transpose. † denotes Hermitian transpose. All column matrices will be shown with underbar. All matrices will be shown in boldface.

The E-field in the 0th guide ($z=0_-$) of height L must equal the E-fields in each of the smaller N guides ($z=0_+$) or vanish on the flat end faces ($z=0_+$) of each of the $N-1$ perfectly conducting septa.

If (2) is a scalar multiplied by the column matrix $\underline{\vec{e}}_0(x)$ and use is made of the orthogonality of the modal fields, then if the product is integrated over the range $0<x<L$, (2) becomes, after algebra

$$\underline{A}_0=[\boldsymbol{H}_{01}\boldsymbol{H}_{02}\boldsymbol{H}_{03}\cdots\boldsymbol{H}_{0N}]\begin{bmatrix}\underline{A}_1\\ \underline{A}_2\\ \underline{A}_3\\ \vdots\\ \underline{A}_N\end{bmatrix}. \tag{3}$$

In (3), the matrices $\boldsymbol{H}_{oi}, i=1,2,\cdots,N$ are the E-field mode-matching matrices whose mnth elements are given by

$$H_{0i,mn}=\frac{\int_{L_i}\vec{e}_{0m}(x)\cdot\vec{e}_{in}(x)\,dx}{\int_L\vec{e}_{0m}(x)\cdot\vec{e}_{0m}(x)\,dx}. \tag{4}$$

The analytical expressions of $H_{0i,mn}, i=1,2,\cdots,N$ for both TE and TM modes are

$$H_{0i,mn}=\begin{cases}\dfrac{n}{L_i}Q_{i,mn}, & \dfrac{m}{L}\neq\dfrac{n}{L_i}\\ R_{i,m}, & \dfrac{m}{L}=\dfrac{n}{L_i}\end{cases}\quad\text{(TE)}\qquad m,n=1,2,3,\cdots$$

$$H_{0i,mn}=\begin{cases}\dfrac{m}{L}Q_{i,mn}, & \dfrac{m}{L}\neq\dfrac{n}{L_i}\\ R_{i,m}, & \dfrac{m}{L}=\dfrac{n}{L_i}\end{cases}\quad\text{(TM)}\qquad m,n=0,1,2,\cdots$$

where

$$Q_{i,mn}=\frac{\dfrac{2\pi}{L}}{\left(\dfrac{n\pi}{L_i}\right)^2-\left(\dfrac{m\pi}{L}\right)^2}\left[(-1)^{n+1}\sin\left(\frac{m\pi}{L}h_i\right)+\sin\left(\frac{m\pi}{L}h_i'\right)\right]$$

$$R_{i,m}=\frac{L_i}{L}\cos\left(\frac{m\pi}{L}h_i'\right),$$

$$h_1=L\quad h_i=h_{i-1}-(L_{i-1}+t_{i-1})\quad h_i'=h_i-L_i.$$

Let us define $\boldsymbol{H}$ as

$$\boldsymbol{H}=[\boldsymbol{H}_{01}\boldsymbol{H}_{02}\boldsymbol{H}_{03}\cdots\boldsymbol{H}_{0N}]. \tag{5}$$

The matrix equation (3) can then be represented more simply as

$$\underline{A}_0=\underline{A}=\boldsymbol{H}\underline{B} \tag{6}$$

where $\underline{B}$ is the generalized E-field mode coefficient column matrix of the N-furcated waveguide.

Turning now to the principle of conservation of complex power at the generalized 2-port, it can be easily shown, using the same reasoning as in [7], that if we assume an arbitrary incident field from the N-furcated guide which can be represented by the column matrix $\underline{B}_+$ and if the scattered fields are represented by $\underline{B}_-$ and $\underline{A}_-$, then the three column matrices are related by the matrix equation

$$\underline{A}_-^\dagger\boldsymbol{P}_A\underline{A}_-=(\underline{B}_+-\underline{B}_-)^\dagger\boldsymbol{P}_B(\underline{B}_++\underline{B}_-) \tag{7}$$

where the matrices $\boldsymbol{P}_A$ and $\boldsymbol{P}_B$ are diagonal and whose diagonal elements are the powers carried by unit amplitude modes in the various waveguides. $\boldsymbol{P}_B$ can be written as

$$\boldsymbol{P}_B = \begin{bmatrix} \boldsymbol{P}_{B1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{P}_{B2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{P}_{BN} \end{bmatrix} \tag{8}$$

where $\mathbf{0}$ is a null matrix. The diagonal submatrices $\boldsymbol{P}_{Bi}, i = 1,2,\cdots N$, have diagonal elements given by

$$P_{Bi,nn} = \frac{L_i}{2} \frac{\left[k_0^2\epsilon_{ri} - \left(\frac{n\pi}{L_i}\right)^2\right]^{1/2}}{\omega\mu_0}, \qquad n = 1,2,3,\cdots \qquad \text{(TE)}$$

$$P_{Bi,nn} = \begin{cases} \dfrac{L_i}{2} \dfrac{\omega\epsilon_0\epsilon_{ri}}{\left[k_0^2\epsilon_{ri} - \left(\frac{n\pi}{L_i}\right)^2\right]^{1/2}}, & n = 1,2,3\cdots \\ L_i\left[\dfrac{\epsilon_0\epsilon_{ri}}{\mu_0}\right]^{1/2}, & n = 0. \end{cases} \qquad \text{(TM)}$$

and

$$P_{A,nn} = \frac{L}{2} \frac{\left[k_0^2\epsilon_{r0} - \left(\frac{n\pi}{L}\right)^2\right]^{1/2}}{\omega\mu_0}, \qquad n = 1,2,3,\cdots \qquad \text{(TE)}$$

$$P_{A,nn} = \begin{cases} \dfrac{L}{2} \dfrac{\omega\epsilon_0\epsilon_{r0}}{\left[k_0^2\epsilon_{r0} - \left(\frac{n\pi}{L}\right)^2\right]^{1/2}}, & n = 1,2,3\cdots \\ L\left[\dfrac{\epsilon_0\epsilon_{r0}}{\mu_0}\right]^{1/2}, & n = 0. \end{cases} \qquad \text{(TM)}$$

Then, if (6) is rewritten so that

$$\underline{A}_- = \boldsymbol{H}(\underline{B}_+ + \underline{B}_-) \tag{9}$$

we can substitute (9) into (7) and after some manipulation, show that the back-scattered column matrix for the N-furcated guide is

$$\underline{B}_- = \left(\boldsymbol{P}_B^\dagger + \boldsymbol{P}_{LB}^\dagger\right)^{-1}\left(\boldsymbol{P}_B^\dagger - \boldsymbol{P}_{LB}^\dagger\right)\underline{B}_+ \tag{10}$$

where

$$\boldsymbol{P}_{LB} = \boldsymbol{H}^\dagger \boldsymbol{P}_A \boldsymbol{H} \tag{11}$$

is the load power matrix of the large guide as "seen" by the N-furcated guide.

Defining the E-field mode coefficient scattering matrix $[S]$ of the generalized 2-port such that

$$\begin{bmatrix} \underline{A}_- \\ \underline{B}_- \end{bmatrix} = \begin{bmatrix} \boldsymbol{S}_{11} & \boldsymbol{S}_{12} \\ \boldsymbol{S}_{21} & \boldsymbol{S}_{22} \end{bmatrix} \begin{bmatrix} \underline{A}_+ \\ \underline{B}_+ \end{bmatrix} \tag{12}$$

we can use (10) and the condition that $\underline{A}_+ = 0$ to obtain

$$\boldsymbol{S}_{22} = \left(\boldsymbol{P}_B^\dagger + \boldsymbol{P}_{LB}^\dagger\right)^{-1}\left(\boldsymbol{P}_B^\dagger - \boldsymbol{P}_{LB}^\dagger\right). \tag{13}$$

Moreover, (9) can be used to show that

$$\boldsymbol{S}_{12} = \boldsymbol{H}(\boldsymbol{I} + \boldsymbol{S}_{22}) \tag{14}$$

where $\boldsymbol{I}$ is the identity matrix.

To determine the remaining two submatrices in (12), we can consider that a field $\underline{A}_+$ is now incident from the left side of the junction, with $\underline{B}_+ = 0$. Then (6) becomes

$$\underline{A}_+ + \underline{A}_- = \boldsymbol{H}\underline{B}_- \tag{15}$$

and the power conservation at $z = 0$ dictates that

$$(\underline{A}_+ - \underline{A}_-)^\dagger \boldsymbol{P}_A(\underline{A}_+ + \underline{A}_-) = \underline{B}_-^\dagger \boldsymbol{P}_B \underline{B}_-. \tag{16}$$

Using (15) in (16) we get

$$(\underline{A}_+ - \underline{A}_-)^\dagger \boldsymbol{P}_A \boldsymbol{H}\underline{B}_- = \underline{B}_-^\dagger \boldsymbol{P}_B \underline{B}_- \tag{17}$$

or

$$(\underline{A}_+ - \underline{A}_-)^\dagger \boldsymbol{P}_A \boldsymbol{H} = \underline{B}_-^\dagger \boldsymbol{P}_B \tag{18}$$

and after taking the Hermitian transpose of (18) and using (15), we can eliminate $\underline{A}_-$ and obtain, with some rearranging

$$\underline{B}_- = 2\left[\boldsymbol{P}_B^\dagger + \boldsymbol{P}_{LB}^\dagger\right]^{-1}\boldsymbol{H}^\dagger \boldsymbol{P}_A^\dagger \underline{A}_+. \tag{19}$$

Then, with $\underline{B}_+ = 0$ in this case, it follows from (12) that

$$\boldsymbol{S}_{21} = 2\left[\boldsymbol{P}_B^\dagger + \boldsymbol{P}_{LB}^\dagger\right]^{-1}\boldsymbol{H}^\dagger \boldsymbol{P}_A^\dagger. \tag{20}$$

Moreover, (15) and (20) can be used to give us

$$\boldsymbol{S}_{11} = \boldsymbol{H}\boldsymbol{S}_{21} - \boldsymbol{I}. \tag{21}$$

It can easily be shown that use of the reciprocity theorem [7], [12] gives the same result for $\boldsymbol{S}_{21}$ as (20), i.e.,

$$\boldsymbol{S}_{21} = \begin{cases} \boldsymbol{P}_B^{\dagger -1}\boldsymbol{S}_{12}^T\boldsymbol{P}_A^\dagger & \text{(by reciprocity)} \\ 2\left[\boldsymbol{P}_B^\dagger + \boldsymbol{P}_{LB}^\dagger\right]^{-1}\boldsymbol{H}^\dagger\boldsymbol{P}_A^\dagger \end{cases} \tag{22}$$

only if the following two conditions are satisfied: 1) the elements of $\boldsymbol{H}$ are real, 2) power orthogonality is satisfied, $\boldsymbol{P}_A$ and $\boldsymbol{P}_B$ are diagonal. These two conditions are satisfied in lossless parallel-plate and rectangular waveguide. However, in the case of lossy waveguide, where the conditions are not satisfied, we can still use (20), or the second formula in (22). The solution is formally exact with matrices of infinite dimension which, for numerical computation, must be truncated. In Section III, numerical results for a variety of junctions are presented, as well as a discussion of the numerical convergence of the CCPT.

III. Numerical Results

A. *Bifurcated Guides*

We consider first the simplest configuration, the junction of a regular parallel-plate waveguide and a bifurcated guide with a centered septum of variable thickness t. Fig. 2 gives the reflection coefficient, when a TE_1 mode is incident from one of the small guides, as a function of L_1/λ and for $t = 0$ and $t = 0.2L_1$. For these calculations, we truncated the various matrices to retain 10 modes in each of the smaller guides and 20 modes in the larger (for $z < 0$). For the case of an infinitely thin spectum ($t = 0$) and $L_1/\lambda < 0.75$, our CCPT results are virtually identical to those given by Lewin [2, p. 282]. The reason for the quantitative disagreement between our results and those of Lewin at high values of L_1/λ is that Lewin's results are based upon a quasi-static analysis.

In Fig. 3, we consider an asymmetrically placed septum with $L_1 = 0.6L$ and for which $t = 0$ and $t = 0.2L$. For the case of a TE_1 mode incident from the large guide ($z < 0$), the magnitude of the reflection coefficient and that of the transmission coefficient from the large guide into guide 1 are plotted in Fig. 3(a) and (b), respectively. In the case of $t = 0$, our results are in good agree-

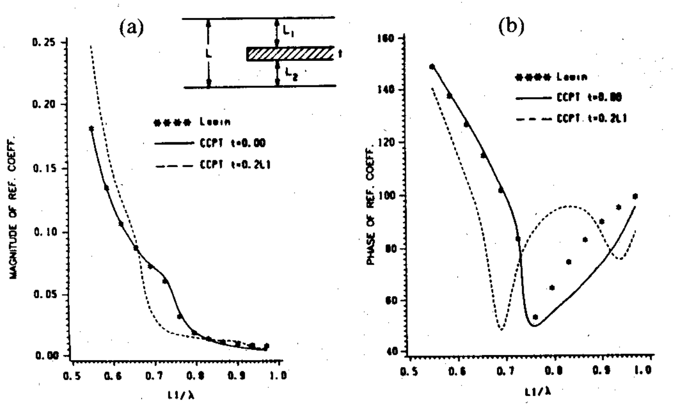

Fig. 2. Reflection coefficient for TE_1 incidence as function of L_1/λ; incidence from one of the small guides: $L_1 = L_2$.

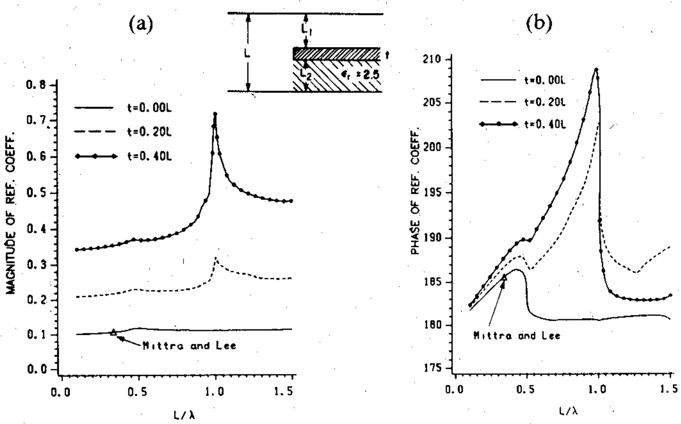

Fig. 5. Reflection coefficient for TEM incidence as function of L/λ; incidence from the large guide, $L_1 = L_2$.

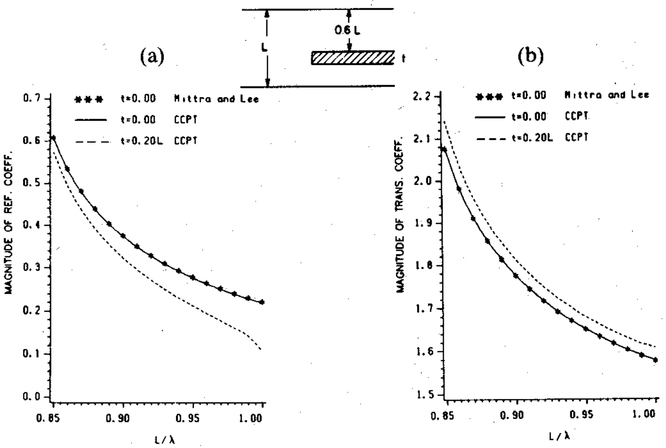

Fig. 3. Magnitude of the reflection coefficient and that of the transmission coefficient from the large guide to guide 1 for TE_1 incidence as function of L/λ.

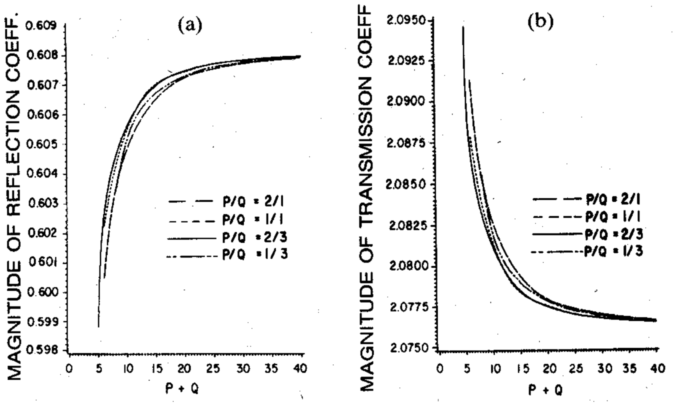

Fig. 4. Convergence of the magnitude of the reflection coefficient and transmission coefficient as a function of $P+Q$ with P/Q as a parameter and for $L_1 = 0.6L$, $t=0$, and $L/\lambda = 0.85$.

ment with those of Mittra and Lee [1, pp. 44–45]. However, in solving this problem using the traditional E- and H-field mode-matching technique and the "direct inversion method" [1, pp. 41], their solutions converged to incorrect values if the ratio of the number P of modes retained in guide 2 and the number Q retained in guide 1 were different from the ratio of guide heights. Only when $P/Q = L_2/L_1$ did the numerical solutions converge to the theoretically exact solutions deduced by other means [1, pp. 45–50]. Mittra and Lee have called this the "relative convergence" phenomenon.

To demonstrate that the CCPT is not affected by the "relative convergence" phenomenon, we consider specific points ($L/\lambda = 0.85$) on the curves of Fig. 3(a) and (b). In Fig. 4(a) and 4(b) are the corresponding convergence curves for four distinct P/Q ratios. In all cases, there is absolute convergence to the analytically correct values. The rate of convergence, however, is greatest when $P/Q = L_2/L_1 = 2/3$. This type of absolute convergence has been demonstrated by Shih and Gray [13] in connection with rectangular-to-rectangular waveguide junctions using a modal analysis technique virtually identical to the CCPT.

With guide 2 filled with a dielectric ($\epsilon_r = 2.5$ and $L_1 = L_2$), and with the thickness t as a parameter, the CCPT solution for the TEM reflection coefficient in the large guide is given in Fig. 5 as a function of L/λ. Mittra and Lee [1] provide a modified residue calculus solution for $t=0$, $L = 0.339\lambda$, and the two solutions agree in magnitude and phase to three significant figures when 20 modes are used in the large guide for the CCPT solution.

B. *Trifurcated Guides*

Treating the microwave 4-port junction of a single parallel-plate waveguide and a trifurcated guide as a generalized 2-port junction, we can, as outlined in Section II, use the CCPT to deduce the scattering matrix of the complete junction.

In order to compare our CCPT results with those of Pace and Mittra [3], we consider first the asymmetrical case where $L_2 = L_3 = 0.5L_1$ and let $t_1 = t_2 = t$ with $t = 0$ (as in [3]) and $t = 0.05L$. The magnitude and phase of the reflection coefficient of the TEM mode, when incidence is from guide 1, are plotted in Fig. 6(a) and (b), respectively, as functions of L/λ; the corresponding results for incidence from guide 3 are given in Fig. 6(c) and (d). Our results agree with those of Pace and Mittra [3] for the $t=0$ case if their reflection coefficient is for guide 1 rather than guide 3.

Fig. 7 illustrates the effect of the dielectric constant and the septum thickness for the case of a trifurcated guide with $L_1 = L_2 = L_3$, $t_1 = t_2 = t$, and $\epsilon_{r1} = \epsilon_{r2} = \epsilon_{r3} = \epsilon_r$. The TEM mode's reflection coefficient for the large guide is given as function of t/L_1 for three different values of ϵ_r. In the case of $t=0$ and $\epsilon_r = 1$, the results of our CCPT solution agree with those given by Pace and Mittra [3]. The convergence results for a typical point in Fig. 7 ($t=0$, $\epsilon_r = 1$) are illustrated in Fig. 8.

C. *4-furcated Guides*

The simplest case of the junction of a regular guide and a guide with three equispaced ($L_1 = L_2 = L_3 = L_4$) septa of equal thickness $t_1 = t_2 = t_3 = t$ is first considered for $t=0$ and $t = 0.08L$. Fig. 9 gives the reflection coefficient for the TE_1 mode in the large guide.

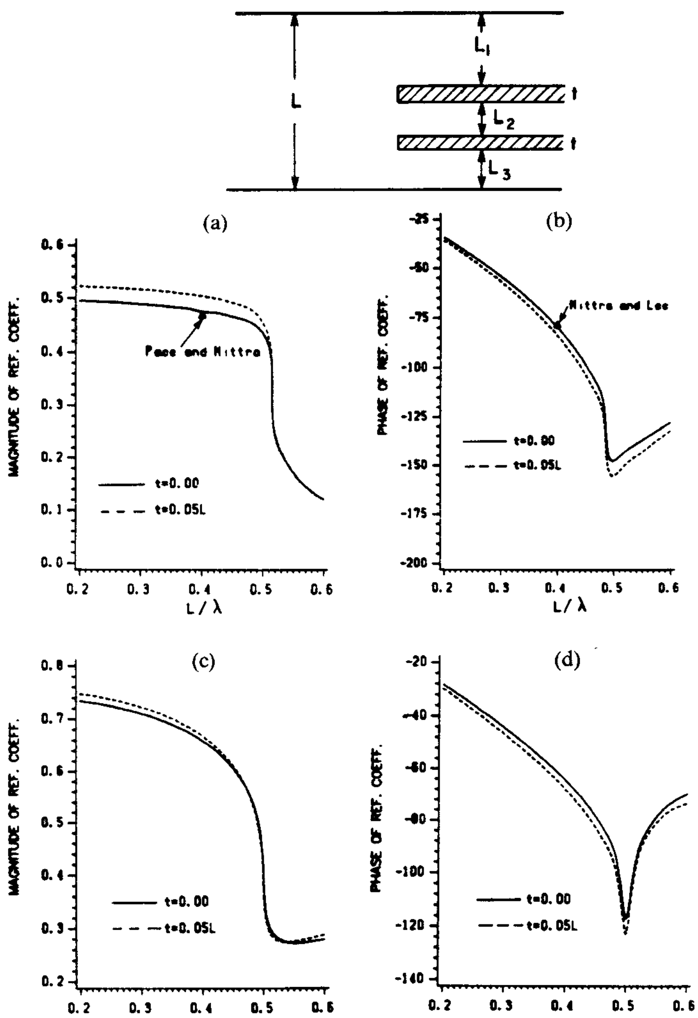

Fig. 6. Reflection coefficient for TEM incidence as function of L/λ: (a),(b) incidence from guide 1; (c),(d) incidence from guide 3; $L_2 = L_3 = 0.05L_1$.

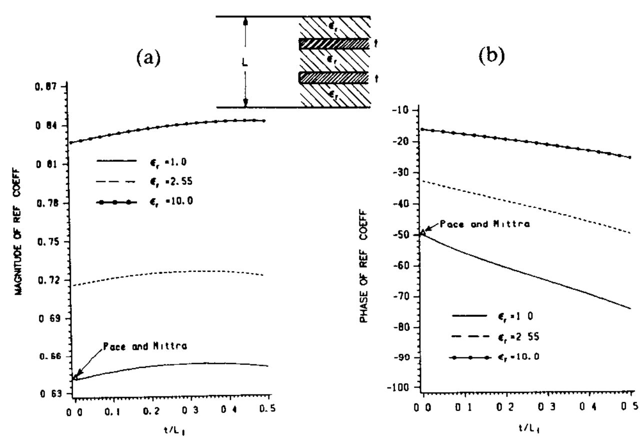

Fig. 7. Reflection coefficient for TEM incidence as function of t/L_1; incidence from the large guide; $L_1 = L_2 = L_3$, $t_1 = t_2 = t$, $\epsilon_{r1} = \epsilon_{r2} = \epsilon_{r3} = \epsilon_r$, $L/\lambda = 0.3$.

Finally, we consider the case of the junction of regular guide and a guide with three arbitrarily spaced septa of arbitrary thickness $L_1 = 0.32L$, $L_2 = L_3 = 0.14L$, $L_4 = 0.12L$, $t_1 = t_3 = 0.07L$, $t_2 = 0.14L$, $\epsilon_{r0} = \epsilon_{r1} = \epsilon_{r4} = 1$, and $\epsilon_{r2} = \epsilon_{r3} = 10$. Fig. 10 gives the reflection coefficients for the TE_1 and TEM modes as functions of L/λ when the incidence is from the large guide. Note that the magnitude of the reflection coefficients for the TE_1 mode in Fig. 10(a) is unity below $L/\lambda = 1.129$, since all small guides are cutoff below this frequency. Just above this frequency, a sharp decrease of the reflection coefficient indicates the start of real power flow into guides 2 and 3. At $L/\lambda = 1.562$, another sharp decrease of the reflection coefficient takes place due to the start of real power flow into guide 1.

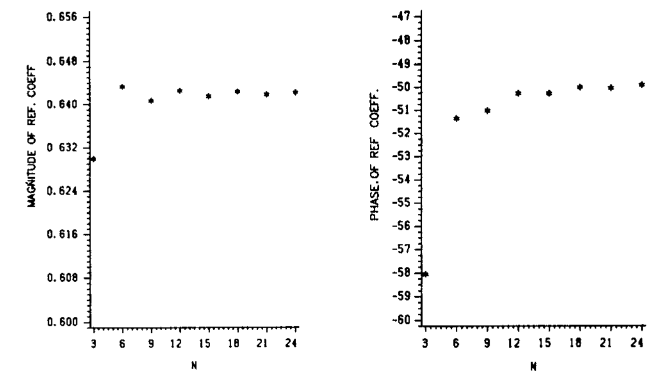

Fig. 8. Convergence results for a typical point ($t/L_1 = 0, \epsilon_r = 1$) in Fig. 7; N is the number of modes in the large guide; the number of modes in each of the small guides is $N/3$.

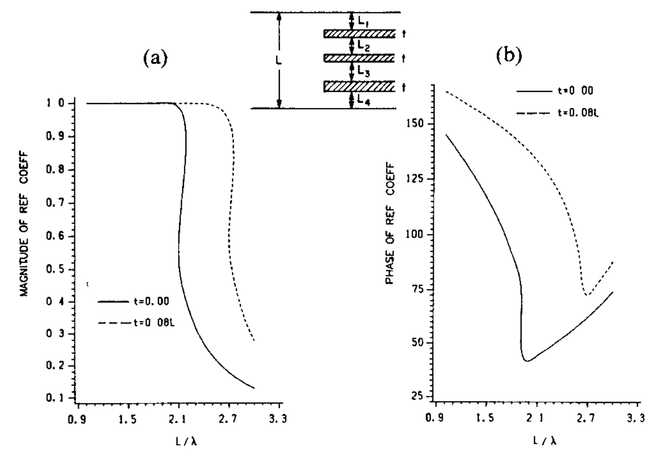

Fig. 9. Reflection coefficient for TE_1 incidence as function of L/λ; incidence is from the large guide; $L_1 = L_2 = L_3 = L_4$, $t_1 = t_2 = t_3 = t$.

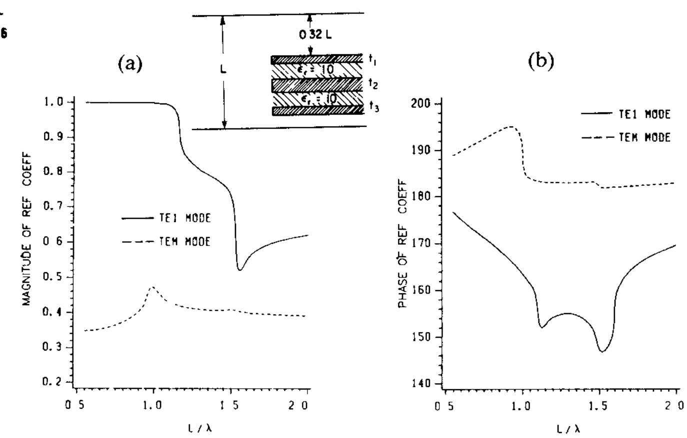

Fig. 10. Reflection coefficient for both of TE_1 and TEM incidence as function L/λ; incidence from the large guide; $L_1 = 0.32L$, $L_2 = L_3 = 0.14L$, $L_4 = 0.12L$, $t_1 = t_3 = 0.07L$, $t_2 = 0.14L$, $\epsilon_{r0} = \epsilon_{r1} = \epsilon_{r4} = 1$ and $\epsilon_{r2} = \epsilon_{r3} = 10$.

IV. Conclusions

This paper has provided a formally exact solution with convergent numerical results to the problem of EM scattering at an N-furcated parallel-plate waveguide junction with arbitrarily spaced thick septa. The problem is formulated so that the complexity of the evaluation procedures is not affected by the number of septa. The calculated results are in excellent agreement with other available data. Moreover, investigation of the effect of matrix truncation indicates that the CCPT solutions converge absolutely to the exact solutions, making the problem of "relative convergence" virtually nonexistent. Possible application of this approach is in the design of *E*-plane filters.

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Microstrip Transmission Line With Finite-Width Dielectric and Ground Plane

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Abstract —**Design data for microstrip transmission lines with finite-width dielectric and ground plane are presented. The characteristic impedance and velocity of propagation are tabulated from results of a moment-method solution of a quasi-TEM transmission-line model of this microstrip structure.**

I. Introduction

A numerical solution for an open microstrip transmission line with a finite-width dielectric and infinite-width ground plane was recently described in a paper by Smith and Chang [1]. This case

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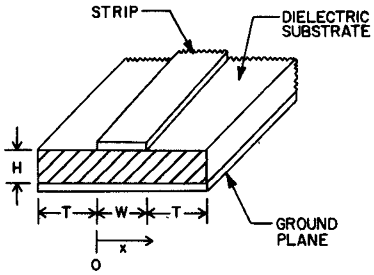

Fig. 1. Microstrip transmission line with both a truncated dielectric substrate and ground plane.

of the truncated dielectric microstrip with an infinite ground plane was considered because it more closely approximates the practical finite substrate case than the idealized infinite-width model normally employed. Consequently, a parameter study of the characteristics of this type of transmission line was presented for design purposes for practical applications.

However, another related model of some importance is that of a microstrip transmission line having both a truncated dielectric and ground plane as shown in Fig. 1. This finite-width dielectric and ground-plane structure represents several practical applications where odd-mode propagation is dominant. One such application of the structure is related to the design of tapered, balanced-to-unbalanced, transformers (baluns) such as that type originally proposed by Duncan and Minerva [2] and later used in principle by Gans, Kajfez, and Rumsey for mode conversion in microstriplines [3]. The resulting transformer employs a tapered transition which has a characteristic impedance that varies continuously in a smooth fashion from the balanced-to-unbalanced transmission line, and the cross-sectional characteristic impedance as a function of length is the desired design quantity in this approach based on the theory of small reflections [4].

A related problem consisting of two perfectly conducting zero-thickness parallel strips of unequal widths in a homogeneous medium has been analyzed to obtain an approximate solution to this class of structures for design purposes [5]. The accuracy of this data is certainly questionable because of the homogeneous modeling of this inhomogeneous structure, particularly for both small T and W/H as defined in Fig. 1. Thus, a numerical solution for the inhomogeneous configuration of Fig. 1 has been developed to obtain a better approximation of line parameters for general design purposes. Tentative results from this numerical analysis indicate that the design data for the homogeneous model is indeed in error by more than ten percent for small W/H ratios [6]. A brief discussion and the computed results of this numerical solution for the truncated dielectric and ground-plane structure are presented in the next section of this paper.

II. Numerical Solution and Results

The transmission-line characteristics for the microstrip problem of Fig. 1 can be obtained using a free-space Green's function formulation in terms of equivalent surface charge sources on the structure boundaries coupled with a moment-method solution for a quasi-TEM model. This approach has been used previously to solve inhomogeneous electrostatic problems, and the theory for this method has been presented in several forms by Smith and Chang [1], Harrington and Pontoppidan [7], Adams and Mautz [8], and Smith [9]. In addition, Rao, Sarkar, and Harrington have recently used this same surface charge formulation to analyze electrostatic fields of conducting bodies in multiple dielectric

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